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Electron Acceleration by Light Waves

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Summary

Light, particularly in the form of evanescent waves, can be employed to accelerate charged particles. Since the field strength in LASER radiation is expected to be of the order of 10^9 V/m, an accelerator of significantly reduced dimensions based upon this concept appears feasible. The proposed accelerator works on what may be defined as the time-reversed Smith-Purcell effect.

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ELECTRON ACCELERATION BY LIGHT WAVES

I. Introduction

Increased physical size, providing longer accelerating paths, has been the trend of modern acceleration design aimed at higher particle energies. An alternative to this approach would be the utilization of greater field strength, if this were feasible.

It is our aim to set forth some ideas which may lead to the realization of this alternative through utilization of the high field strength attainable in optical MASER radiation.

The mechanism through which charged particles may be accelerated by light waves is related to an effect reported by Smith & Purcell.[1] This effect has been demonstrated to produce radiation in the visible and infrared region as a result of an electron traveling near the surface of a diffraction grating. The structure of the grating causes periodic motion of the charge induced on the surface by the moving electrons which in turn gives rise to radiation of frequency ν . This frequency is related to the electron velocity v and the grating pitch δ . According to grating manufacturers intense radiation sources based on this effect are now feasible. This implies the existence of gratings of sufficient flatness, controlled grating spacing and groove geometry to permit an electron beam to pass within maybe 1/2

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the grating pitch over a reasonably extended grating comprising 50,000 grooves or more as reported by Smith & Purcell.

Now imagine the process reported by Smith & Purcell to occur in reverse. Light is incident upon the grating surface, and the electric field of the light wave will accelerate the electron. Since an electric field strength of 10^9 V/m in the radiation from an optical MASER is feasible, the reverse of the Smith-Purcell effect could be the basis for an unusually short accelerator.

To complete this introduction, attention is drawn to further considerations based on Toraldo di Francias'[2] explanation of the Smith-Purcell effect. His theory shows that the grating may consist equally well of a dielectric material and explains the role played by evanescent waves in the Smith-Purcell effect. Both of these concepts are more fully treated in what is to follow.

II. Acceleration by Generalized Plane Waves

The acceleration of electrons by light waves can best be described by considering two conditions which must be fulfilled: (1) the phase velocity V of the accelerating field must match the particle speed v where $v < c$ always holds; and (2) the electric field must have a component in the direction in which the particles travel.

Without loss of generality, we consider only plane waves moving parallel to the particle.

A monochromatic plane wave, moving in the z direction with phase velocity V, not dependent on the Y coordinate has the form:

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$$U(x,z,t) = f(x)\cdot \exp[-i\omega(t - z/v)] \tag{1}$$

In a charge-free dielectric and homogeneous space, the wave equation

$$(\nabla)^2 U - (n/c)^2 \cdot \partial^2 U/\partial t^2 = 0 \tag{2}$$

then reduces to

$$d^2f/dx^2 + f\cdot(n\omega/c)^2\cdot\{1-(c/nV)^2\} = 0 \tag{3}$$

For convenience of notation let the term in brackets

$$\{1-(c/nV)^2\} = \alpha \tag{4}$$

If $\alpha = 0$ one obtains the usual plane wave, $f =$ constant and $V = c/n$ but without a longitudinal component of the electric field.

If $\alpha > 0$ one obtains a plane wave with a transverse modulated amplitude $f(x) = \cos[(n\omega/c)\cdot\alpha^{1/2}\cdot x]$. Matching of the phase velocity V and the particle speed v is possible, since $\alpha > 0$ satisfies the condition for a phase velocity lower than the vacuum velocity of light, i.e. $c > V > c/n$. Moreover, this requires the particle beam to exist in the presence of matter in order to have a refractive index $n > 1$.

From Maxwell's divergency equation follows the existence of a longitudinal component in the electric field. These transverse modulated

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plane waves should thus be able to accelerate electrons.

This was pointed out already by K. Shimoda[3] who proposed a gas filled cavity surrounded by a cylindrical MASER as an accelerator. He noted also, that his idea predicts the reverse process of the Cerenkov effect. This is indicated by the velocity condition $v = V > c/n$ and it follows immediately if one composes the transverse modulated plane wave by two symmetrically inclined ordinary plane waves:

$$2\cos[(n\omega/c)\,\ell^{1/2}\cdot x]\ \exp[-i\omega(t-z/v)] =$$

$$\exp[ik(\alpha x+\gamma z)-\omega t] + \exp[ik(-\alpha x+\gamma z)-\omega t];\quad k = n\omega/c,\quad \alpha^2+\gamma^2=1$$

$$k_z = \omega/v = k\gamma = k\cos\varepsilon\ ;\quad \cos\varepsilon = c/nv$$

The connection with the Cerenkov effect is evident, since the inclination ε of these two ordinary plane waves is given by the MACH formula.

III. Acceleration by Evanescent Plane Waves

If $\ell < 0$ or $V < c/n$ the differential equation for the wave amplitude f(x) (eq. 3) has an exponential solution.

This leads to an evanescent wave of the form

$$U(x,z,t) = \exp[-k''x]\cdot\exp[i(k'z-\omega t)]$$

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This is the only kind of a plane wave with a phase velocity V smaller than c even in vacuum $(n = 1)$. In other words, this seems to be the only way to match the speed of the light phase with that of the particle when the process is to take place in vacuum.

IV. Means for Producing Evanescent Waves

Before proceeding with the discussion of the properties of evanescent waves, let us consider means for their production.

The existence of these waves has been predicted already by Lord Rayleigh in connection with Woods anomalies in grating diffraction. Under certain conditions the diffraction grating converts ordinary plane waves into evanescent waves. For their existence, it is necessary to have a wave vector k smaller than one of its components: $k < k_z$. The law of diffraction and refraction, which governs the tangential components of the wave vector at a boundary, as in Fig. 1, tells us when this will happen: $\sin\varphi' = n \cdot \sin\varphi + m\lambda/a$

(m = diffraction order)

$$k_z = k \sin\varphi'; \quad |k_z| > k \rightarrow |\sin\varphi'| > 1$$

if $m = 0$, $n > 1/\sin\varphi$ (total reflection)

if $m \neq 0$ and $\varphi = 0$, $a < |m| \cdot \lambda$ (fine grating).

This indicates that total reflection and diffraction (not only at periodic objects and not only at normal incidence ($\varphi = 0$) can convert ordinary plane waves into evanescent waves. The dimension of the zone near the boundary, where the amplitude assumes large values, is given by the damping

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length $\lambda'' = \lambda/(\sin^2\varphi' - 1)^{1/2}$. The phase velocity is $V = c/\sin\varphi'$.
Here again one sees that the damping length goes to infinity, if the phase velocity and with it the speed of the particle approaches the speed of light in vacuum. This is of practical importance since the electron must travel in close proximity of the grating where the amplitude of the electric field is not seriously damped. Hence, the difficulty of guiding the electron becomes less critical already when the energy is several times higher than the rest energy.

V. Vectorial Properties of Evanescent Waves

One can consider evanescent waves as plane waves, having a complex wave vector:

$$k = k' + ik''; \quad k^2 = (n\omega/c)^2 = k'^2 - k''^2; \qquad k' \perp k''$$

The wave equation tells us that the real and imaginary parts are normal to each other, e.g. in the z and x direction, respectively. The real part is longer than the wave vector $k' > k = n\omega/c$ of an ordinary plane wave. From that follows the inequality for the phase velocity

$$V = \omega/k' = \omega/k \cdot k/k' = c/n \cdot k/k' < c/n$$

One can define a damping length $\lambda'' = 2\pi/k''$ describing the attenuation of the amplitude along the x axis. This damping length λ'' is connected with the wavelength $\lambda' = 2\pi/k'$ and with the velocity $V = v = c \cdot \beta$

$$\lambda''/\lambda' = 1/(1-\beta^2)^{1/2}; \quad \lambda' = \beta\lambda; \quad (n=1)$$

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This indicates that there will be essentially no damping if one approaches the speed of light in vacuum $\beta \to 1$

The vectorial properties of the evanescent waves follow from Maxwell's divergency equation; that is, the longitudinal component E_z is associated with the transverse component E_x as $ik' E_z = k'' E_x$ or

$$|E_z| = |E_x|\,(k''/k') = |E_x|\,(1-\beta^2)^{1/2}$$

$$E_{\substack{x\\z}} = A_{\substack{x\\z}} \exp[-k''x]\cdot \exp[i(k'z-\omega t)]$$

Hence, the longitudinal component E_z which produces the acceleration of the electrons will decrease when one approaches the vacuum speed of light.

The significance of this problem was pointed out by W. Panofsky;[4] possible means of its avoidance will be the subject of a future communication.

VI. Focussing and Phase Stability of the Particle Beam

The associated transverse component E_x will cause the electron to deviate from the desired path or cause defocussing of the electron beam. This is true also if one superposes two evanescent waves with opposite directions of damping. This means a replacement of the exponential amplitude function $f(x) = \exp[-k''x]$ by the hyperbolic cosine $\cosh[k''x]$. For this case the field lines are shown in Fig. 2. There, at $z - vt = \lambda'/2$ is a longitudinal saddle which can hold the electron at the phase velocity of the wave and thus provide acceleration. Since Maxwell's divergency equation forbids a crater but allows only saddles, one cannot provide phase stability and focus at the same time. If after a period of acceleration, the electron

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is deviated too much towards the grating, we let it change into a transverse saddle (at $z - vt = \lambda'$). Indeed the field will have such a jump of $\lambda'/2$ if it arrives at a point above the grating where the spacing changes by half a period (see Fig. 2, far right). This phase jump has been demonstrated experimentally in the case when ordinary plane waves emerge from the grating.[5]

The transverse component is always greater than the longitudinal component of the electric field. Therefore, the distance required to re-focuss the electron, i.e. the path length over which the electron rides in the transverse saddle, is a relatively small fraction of the grating length.

After a short distance, another jump in the grating will transfer the electron again to an accelerating saddle. Actually this proposal is nothing more than the implementation of Courant, Livingston, and Snyder's idea of the alternating gradient in this type of accelerator.

VII. Realization of Particle Acceleration by Light Waves

It may be somewhat premature to consider the problems of practical realization of an accelerator based upon the ideas here set forth. However, their early statement may lead to timely solutions.

The problem of guiding the electrons properly is certainly a delicate one. Since these difficulties seem to have been resolved in the case of Smith-Purcell radiation sources, they should not be an obstacle in the case of an accelerator based upon the inverse process.

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Especially for high speed electrons the alignment may become easier, since the transverse damping length λ'' of the accelerating field increases by a factor of $\beta(1-\beta^2)^{1/2}$.

The phase of the accelerating field has to be stable for the time of interaction, which may be about 10^{-9} sec. for a 30 cm grating. This is feasible for light generated by an optical MASER.

Repetitive interaction with a grating by virtue of a circular path appears difficult to realize in terms of phase stability at optical wave length over many revolutions. This possibility should not be ruled out for longer wave length.

Selection of a suitable mode for the optical MASER is a problem which seems to be on the way to a solution.

The amount of power within the illuminating beam has to be large enough to overcome the energy losses of the electrons due to Smith-Purcell radiation. Therefore, the power output of the LASER limits the possible electron current.

The amount of current is also related to the question, how effectively the grating may convert incident ordinary plane waves into evanescent waves. This question has some similarities with frustrated total reflection and is analogue to the problem of the blaze of a grating, where it is intended to convert all of the incident light into a single diffraction order. It is likely that a very low angle of incidence will be most favorable, since the Smith-Purcell radiation of very fast electrons emerges for the most part along and nearly parallel to the direction of electron acceleration (see Toraldo, Ref. 2). This also appears reasonable if one thinks in terms of transitions of momentum.

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The possible monochromaticity in speed $\Delta V/V$ can be deduced from the uncertainty principle: $\Delta z \cdot \Delta k_z \sim 2\pi$; $k_z = 2\pi/\lambda'$;

$$V = \lambda' \cdot \nu = \beta \cdot c \quad ; \quad \Delta V/V \sim \lambda'/\Delta z = \beta \cdot \lambda/\Delta z$$

where Δz denotes the path length along the grating. It is clear that the length of the accelerator is inversely proportional to the field strength to attain a given acceleration. The feasible field strength depends not only upon the output of the source and geometrical configuration, but may be limited by breakdown effects. Little is known about breakdown effects at optical frequencies, and as A. L. Schawlow pointed out, this problem may here be of significance.

Addendum

Relating to the question of the vanishing field E_z when $\beta \to 1$ it can be stated that this is true in the case of "one-side-interaction" like in Fig. 3, but not in the case of "both-side-interaction" (Fig. 4). Hence the accelerators feasibility extends to $\beta \to 1$.

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FIGURES

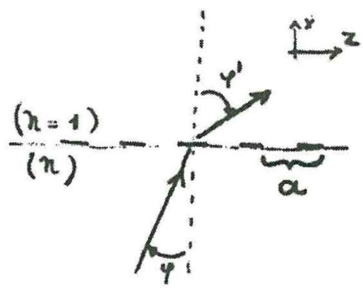

Figure 1: Diffraction and refraction of light at a plane surface, containing a grating.

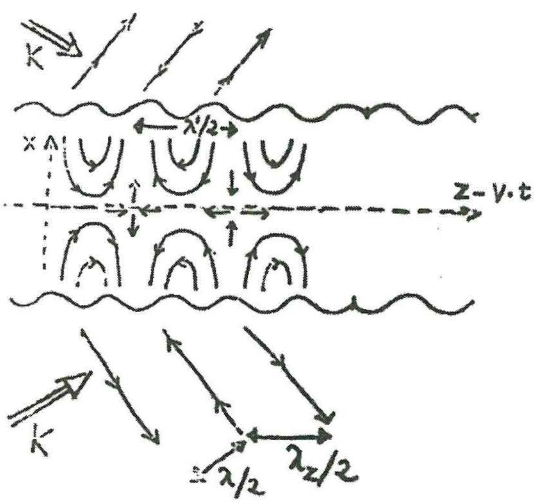

Figure 2: The wavy lines represent the surfaces of two transparent gratings. The electron is traveling along the Z-axis. From the outside arrive two ordinary plane waves, assumed to be emitted by the same source. They are (partially) converted into a pair of evanescent waves. Straight lines with arrows represent the electric field of the waves. There is a longitudinal saddle at $Z - vt = \lambda/2$ which is suitable for acceleration, since it forces the electron to stay on this traveling saddle. But this saddle has transverse gaps. Hence at $Z - v \cdot t - \lambda'$, it is suitable for transverse focussing. At the far right both gratings have a phase jump by half a period. This shall force the electron to change from a longitudinal to a transverse saddle or vice versa. Note that the following inequalities will hold:

$$\lambda_z \geq \lambda \gtrless \lambda' ; \quad a \gtrless \lambda'$$

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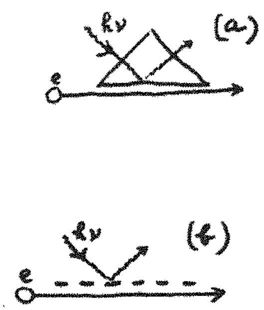

Figure 3:
"One-side-interaction"

(a) one prism for producing the evanescent waves.

(b) one grating.

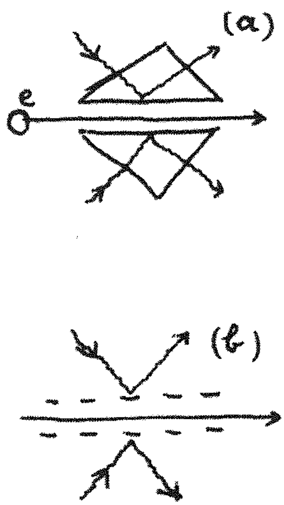

Figure 4:

"Two-side-interaction"

(a) the particle passes between two prism

(b) between two gratings. Now E_z does not tend to zero if β approaches 1.

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